An Effective Approach for Study of Multiple Discontinuities of Transmission Lines

Yansheng Xu and Renato G. Bosisio, *Senior Member, IEEE*

Abstract—**In this paper, a novel approach for calculation of discontinuities of transmission lines is presented. This approach is flexible, simple and effective. For calculation of multiple discontinuities or taking into account the thickness of the obstacles, it is only necessary to transfer the relationship between the electric and magnetic field components from one discontinuity to another and match them on the last one. The method of transfer may be arbitrary, it may also be realized by using the well-known method of lines or others methods. Both single and multiple waveguide discontinuities are calculated and the computed results are in good agreement with the literature. Examples of finite thickness waveguide discontinuities are also given. The proposed method may be readily used to calculate microstrip discontinuities. Extension to discontinuities of other types of transmission lines can also be performed.**

I. Introduction

ALTHOUGH THERE are a great number of methods to analyze the discontinuity problems of waveguides and transmission lines, the problems of multiple discontinuities and finite thickness discontinuities are still very complicated to analyze. One method is to calculate the isolated individual discontinuity and neglect the influence from all others and then consider the interactions between them [1], [2]. This method is quite complicated when the distance between the discontinuities are close and the contributions of higher order modes cannot be neglected. Another method of calculation is to simulate all the discontinuities as a whole. This method will lead to a complicated 3-D problem which is often very time-consuming. To facilitate the solution of this problem, a novel approach is proposed in which the field relationship is transfered from one discontinuity to another and matched only at the last discontinuity. This approach appears rather simple and easy to handle and it is described below.

II. Theory

The proposed approach is based on the concept of making calculations from one end of the transmission line to the other. The relationship between the tranverse field components is calculated at the first discontinuity at one end of the transmission line and then transfered from one discontinuity to the next. In this way we arrive at the last discontinuity, at the other end of the transmission line, and match the field components there. In principle, the transmission line is not necessarily a straight one, it may be curved or it can have other shapes with the only restriction that the above transfer be numerically completed. The modeled transmission line may also be arbitrary, it may be a section of waveguide, microstrip line or other planar/nonplanar lines. The numerical method in the procedure transfer may also be different and it may be chosen as is found convenient.

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The authors are with Groupe de Recherches Avancées en Microondes et en Électronique Spatiale, (POLY-GRAMES), Département de Génie Électrique et de Génie Informatique, École Polytechnique de Montréal, Montréal, Québec, Canada H3C 3A7.

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1) General Formulation: In general, the procedure of solution may be formulated as follows. For simplicity we assume that only the dominant modes can propagate in the first section (I) and the last section $(N+1)$ and in the other sections this restriction does not exist. At first we fix the folowing relationship of transverse field components at the input port 1 in Fig. 1. We assume that only the dominant mode exists at this port and hence it is easy to obtain the following equation

$$E_{1t} = [A_1]H_{1t}. \tag{1}$$

After the transfer through the first section I to plane 2, we have

$$E_{2t} = [A_2][A_1]H_{2t}. \tag{2}$$

By this way we may transfer this relationship from one plane to another and finally arrive to the last plane n and we have

$$E_{nt} = [A_n][A_{n-1}]\cdots[A_2][A_1]H_{nt}. \tag{3}$$

It is assumed that the length of section $N+1$ is long enough to attenuate all the higher order modes and only the dominant mode can exist at plane $n+1$ and we then have

$$E_{n+1,t} = [A_{n+1}]H_{n+1,t}. \tag{4}$$

Transferring this relationship back to the plane n we have an additional field equation

$$E_{nt} = [A'_n][A_{n+1}]H_{nt}. \tag{5}$$

A deterministic equation may be obtained from (3) and (5)

$$[A_n][A_{n-1}]\ldots[A_2][A_1] - [A'_n][A_{n+1}] = 0. \tag{6}$$

Solving this equation we may obtain the needed S-parameters of the cascaded discontinuities. The required line lengths of the first section I and last section $N+1$ may be determined from the propagation constants of the lowest higher modes of these sections.

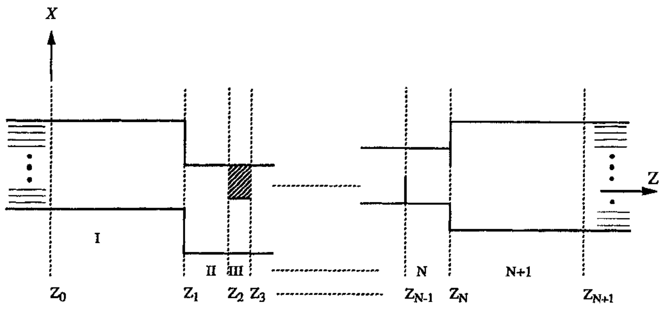

Fig. 1. Illustration of a section of rectangular waveguide, containing multiple discontinuities.

2) Examples of rectangular waveguide discontinuity problems: In the following we choose the well-known method of lines to analyse some rectangular waveguide discontinuity problem as an example. The waveguide is assumed to be empty, that is filled only with air. It is easy to extend this case to other more complicated, dielectric filled systems. As shown in Fig. 1, the axis of the waveguide is assumed to be along the z-axis. The time dependence of the fields $e^{j\omega t}$ is suppressed everywhere in this paper. For the E-plane discontinuity excited by TE_{10} waves, the following relations exist between the field components and the potential Ψ

$$E_x = 0 \qquad H_x \sim \Psi$$
$$E_y \sim \frac{\partial \Psi}{\partial z} \qquad H_y \sim \frac{\partial^2 \Psi}{\partial x \partial y}. \tag{7}$$

Since the discontinuities along the y-axis are absent, the dependence of the potential on the y-axis may be described by $\sin(\pi y/a)$, where a is the dimension of the waveguide along y-axis and we need only to make the ratio of $\frac{\partial \Psi}{\partial z}/\Psi$ continuous at the discontinuities. The potential Ψ is governed by the following differential equation

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \epsilon_0 \Psi = 0. \tag{8}$$

In our calculation, the entire waveguide is divided into N subregions with $(N-1)$ discontinuities and the boundaries of the subregions should contain the surfaces of the discontinuities as shown in Fig. 1. After discretization according to the standard procedure in [4] with the discretized potential line along the z-axis and transformation to the spectral domain, we have an uncoupled ordinary differential equation of the transformed potential $\vec{\Phi}_i$ in each subregion

$$\frac{\partial^2 \vec{\Phi}_i}{\partial z^2} - \vec{k}_i^2 \vec{\Phi}_i = 0 \tag{9}$$

for $i = 1, 2, 3, \ldots .N$.
The discretization distance h is kept constant in all subregions of the waveguide to garanty the convergence of the calculation [5]. The calculation should begin from the first subregion with its left boundary tending to minus infinity. Since there exists only one mode which can propagate and all higher modes attenuate along the input and output port of the waveguide, we may choose a plane with $z = z_0$ at a distance far enough from the first discontinuity and consider that only the dominant mode exist at this plane and we have

$$d\vec{\Phi}_1/dz = -j\frac{\beta_1(1+r)}{(1-r)}\vec{\Phi}_1 \tag{10}$$

at $z = z_0$ where β_1 is the propagation constant of the dominant mode in subregion I, r is the reflection coefficient of the same mode and $\vec{\Phi}_1$ is the solution of (9) of the first subregion I.
The next step is to transfer the potential ratio $(d\vec{\Phi}_i/dz)/\vec{\Phi}_i$ from $z_0, z_1, \ldots$ to z_N. To this end we should use the following formula of transfer within the same subregion

$$\frac{d\vec{\Phi}_i}{dz}\,|_{z=z_i} = H_i \cdot \vec{\Phi}_i\,|_{z=z_i} \tag{11}$$

with

$$H_i = [\vec{k}_i \cdot \sinh(\vec{k}_i d_i) + \cosh(\vec{k}_i d_i) \cdot H_{i-1}] \cdot \left[\cosh(\vec{k}_i d_i) + \frac{\sinh(\vec{k}_i d_i)}{\vec{k}_i} \cdot H_{i-1}\right]^{-1} \tag{12}$$

and

$$\frac{d\vec{\Phi}_i}{dz}\,|_{z=z_{i-1}} = H_{i-1} \cdot \vec{\Phi}_i\,|_{z=z_{i-1}} \tag{13}$$

where $d_i = z_i - z_{i-1}$. By this way, we may transfer the relationship between the potential and its derivative of the same subregion i from $z = z_{i-1}$ to $z = z_i$. At each boundary $z = z_i$ we should also establish the relationship between the potential and its derivatives of the next subregion $(i+1)$ from the matrix $H_i|_{z=z_i}$. This procedure will be discussed in the next section. By this way we can transfer from one subregion to another and arrive at the boundary $z = z_N$ of the subregion N with the field matrix H_N. The field matrix of the last subregion $N+1$ at cross section $z = z_{N+1}$ which is far enough away from the last distinuity N and hence all the higher order modes are attenuated and only the dominant mode exists. Therefore we have

$$d\vec{\Phi}_{N+1}/dz\,|_{z=z_{N+1}} = -j\beta_2 \vec{\Phi}_{N+1}\,|_{z=z_{N+1}} \tag{14}$$

where β_2 is the propagation constant of the dominant mode of the output port waveguide.
By using (12) we may transfer this relationship to the boundary $z = z_N$ and find out the matrix $H_{N+1}\,|_{z=z_n}$. The boundary conditions at the last boundary may be fulfilled by using the following formula

$$(H_N - H_{N+1})\,|_{z=z_n} = 0 \tag{15}$$

and the reflection constant r can readily be solved from this deterministic equation.

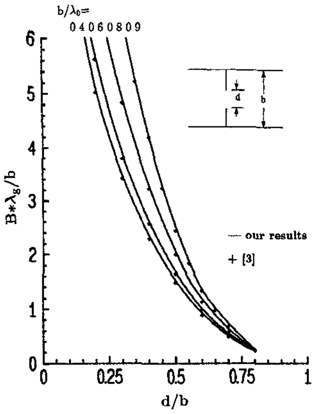

Fig. 2. Dependence of the normalized susceptance of a diaphragm in a rectangular waveguide on the ratio d/b with b/λ_0 as a parameter, ———— our results, + + ++ data from reference [3]

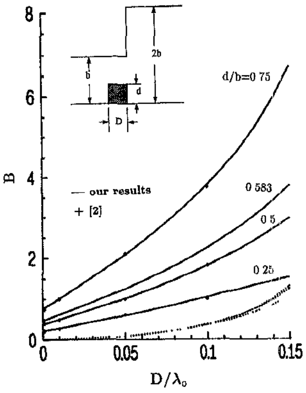

Fig. 4. Dependence of the normalized susceptance of a step with a finite thickness diaphragm in a parallel plate waveguide on the ratio D/λ_0 with $b = 0.1\lambda_0$ and d/b as a parameter, ———— our results, + + ++ data from Ref. [2]. . . the imaginary part obtained from the solution of the normalized susceptance B.

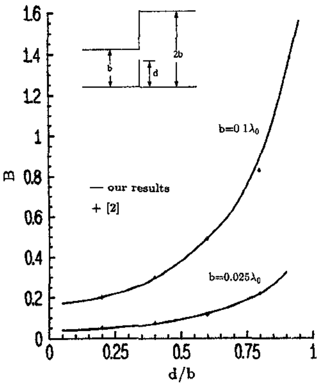

Fig. 3. Dependence of the normalized susceptance of a step with a diaphragm in a parallel plate waveguide on the ratio d/b with b as a parameter, ———— our results, + + ++ data from Ref. [2]

3) Fulfillment of boundary conditions on the metallic walls at the discontinuities: In order to establish the relationship between the potential and its derivatives of the subregion $(i+1)$ from the matrix $H_i|_{z=z_i}$ of the previous subregion i at the plane of discontinuities $z = z_i$, the boundary conditions at the discontinuities should be fulfilled. To this end, we should remember that the boundary conditions at the metallic surface require $d\Psi/dz = 0$ and at the slot the matrix H_i continuous and all this should be completed in the original domain. Let us consider the general case of displacement of two waveguides 1 and 2 with different widths in which s is the common aperture of these two waveguides, s_1 and s_2 are respectively that parts of apertures of waveguides 1 and 2, short circuited by metallic walls. It is clear that at apertures s_1 and s_2 we have $d\Psi/dz = 0$. The first step is to take the inverse of H_i and transform it back into original domain

$$\vec{\Psi}_i\Big|_{z=z_i} = G_i \cdot \frac{d\vec{\Psi}_i}{dz}\Big|_{z=z_i}. \tag{16}$$

The next step lies in splitting the $G_i|_{z=z_i}$ matrix into two parts, corresponding to the aperatures s and s_1 of waveguide 1

$$\begin{pmatrix} \vec{\Psi}_{i,s} \\ \vec{\Psi}_{i,s_1} \end{pmatrix}\Bigg|_{z=z_i} = \begin{pmatrix} G_{i,s} & G_{i,ss_1} \\ G_{i,s_1s} & G_{i,s_1} \end{pmatrix} \cdot \begin{pmatrix} \frac{d\vec{\Psi}_{i,s}}{dz} \\ 0 \end{pmatrix}\Bigg|_{z=z_i} \tag{17}$$

and hence at the common aperture s

$$\vec{\Psi}_{i,s}\Big|_{z=z_i} = G_{i,s} \cdot \frac{d\vec{\Psi}_{i,s}}{dz}\Big|_{z=z_i}. \tag{18}$$

Since the fields are continuous at the common aperture s, (18) is also valid for subregion $i+1$ at $z = z_i$. The next step is to construct the relationship between potential $\vec{\Psi}$ and its derivative for subregion $i+1$ at $z = z_i$

$$\begin{pmatrix} \frac{d\vec{\Psi}_{i+1,s}}{dz} \\ 0 \end{pmatrix}\Bigg|_{z=z_i} = \begin{pmatrix} F_{i+1,s} & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \vec{\Psi}_{i+1,s} \\ \vec{\Psi}_{i+1,s_2} \end{pmatrix}\Bigg|_{z=z_i}$$
$$= F_{i+1} \cdot \begin{pmatrix} \vec{\Psi}_{i+1,s} \\ \vec{\Psi}_{i+1,s_2} \end{pmatrix}\Bigg|_{z=z_i} \tag{19}$$

with $F_{i+1,s} = [G_{i,s}]^{-1}$. The needed matrix $H_{i+1}|_{z=z_i}$ is obtained by transformation of matrix $F_{i+1}|_{z=z_i}$ into the spectral domain.

4) The proposed method of simulation may be readily extended to the microstrip discontinuities by using the waveguide model [6]. It is also possible to solve the 3D problem of microstrip discontinuities through two dimensional discretization at each plane of discontinuities and perform the above procedures.

One advantage of this approach is that it is easy to formulate and the computer algorithm becomes quite simple. Another advantage lies in that the order of the matrix in the final determinantal equation is independent on the number of discontinuities and hence the simulation time increases insignificantly in the case of many discontinuities.

III. Numerical Results

Some examples of numerical calculation for E-plane discontinuities in waveguides are performed by using the above described method. In Fig. 2 a diaphragm in a rectangular waveguide is simulated and the results of the normalized

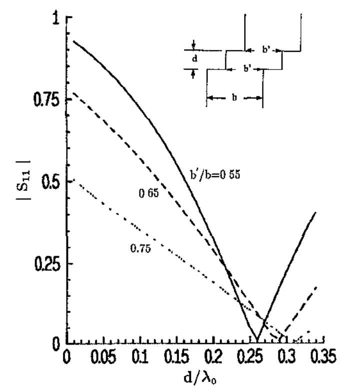

Fig. 5. Dependence of the magnitude of S_{11} on the distance d between two displacements of a rectangular waveguide with the displacement b' as a parameter and $b = 0.3\lambda_g, a = 0.8\lambda_0$.

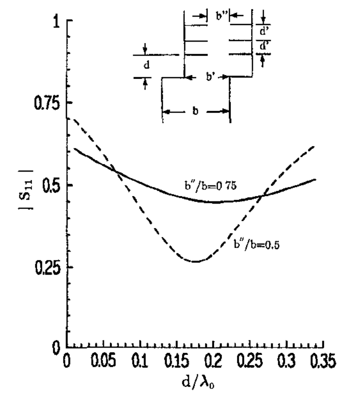

Fig. 7. Dependence of the magnitude of S_{11} on the distance d between a displacement and three diaphragms in a rectangular waveguide with b''/b as a parameter and $b = 0.3\lambda_g$, $b'/b = 0.5, a = 0.8\lambda_0, d' = 0.05\lambda_0$.

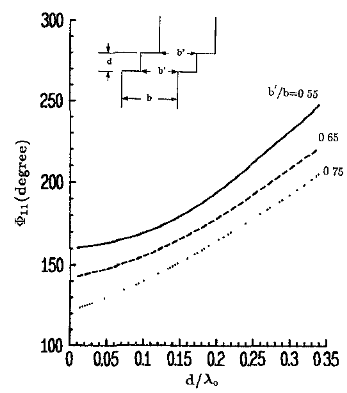

Fig. 6. Dependence of the phase of S_{11} on the distance d between two displacements of a rectangular waveguide with the displacement b' as a parameter and $b = 0.3\lambda_g$, $a = 0.8\lambda_0$.

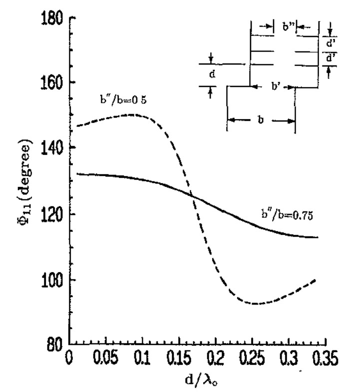

Fig. 8. Dependence of the phase of S_{11} on the distance d between a displacement and three diaphragms in a rectangular waveguide with b''/b as a parameter and $b = 0.3\lambda_g, b'/b = 0.5, a = 0.8\lambda_0, d' = 0.05\lambda_0$.

shunt susceptance B are found to be in good agreement with the literature [3]. The data for a step with an infinite thin diaphragm in a parallel plate waveguide are shown in Fig. 3 and Fig. 4 contains a finite thickness diaphragm with comparisons to reference [2]. Good agreement is seen from the results. The dotted lines in Fig. 4 refer to the imaginary part obtained from the solution of the normalized susceptance B. This part is not small and cannot be neglected when d/λ_0 is greater than 0.1. This means that in such cases the discontinuities cannot be represented by a single shunt susceptance. The results of double displacements in a rectangular waveguide with dimensions $a \times b$ are shown in Figs. 5 and 6. In Fig. 5 the magnitudes of S_{11} depend on the distance between these two displacements d and tend to zero when $d \simeq 0.25\lambda$. This may be explained by superposition of the reflected waves from these two displacements. The results of a displacement and three diaphragms in a section of a rectangular waveguide are shown in Figs. 7 and 8.

IV. Conclusion

A novel approach for calculation of multiple and finite thickness discontinuities in transmission lines is presented. It is simple, flexible, and easy to handle. By using this approach, the computer algorithm is simplified and the numerical modeling becomes more efficient. This feature is more evident when a large number of discontinuities are simulated. The effectiveness of this approach is proved by numerical calculations of multiple discontinuities in waveguides. The proposed approach may readily be used to calculate microstrip discontinuities using the waveguide model. Extension to discontinuities of other types of transmission lines can also be realized.

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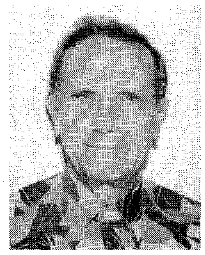
Yansheng Xu graduated from Tsing Hua University, Beijing, China in 1952 and received the degree of Candidate of Technical Science from the Institute of Radio Physics and Electronics, Academy of Science, Moscow, Russia in 1961.

He joined the Beijing Institute of Radio Measurements and engaged in radio communications and radar techniques. He is now with Centre de Recherches Poly-GRAMES at Ecole Polytechnique de Montreal, Montreal, Canada. His main research interests include methods of analysis and computer-aided design of MMIC's and MHMIC's, microwave measurements, microwave ferrite devices and chiral materials. He has published over 80 technical papers in his field of interest.

Mr. Xu is Fellow of the Chinese Institute of Electronics (CIE).



Renato G. Bosisio (M'79-SM'89) received the B.Sc. degree from McGill University, Montreal, Canada, and the MSEE degree from the University of Florida, Gainesville, USA.

He has been engaged in microwave research and development work with various firms: Marconi and Varian in Canada, Sperry in the U.S., and English Electric in England. He is presently head of PolyGRAMES, a university microwave research center at Ecole Polytechnique de Montreal, Montreal, Canada, where he teaches microwave theory and techniques. He is actively engaged in six-port technology, dielectric measurements, and computer-aided testing and design of active and passive microwave devices in MH+MIC.